P.G. Sem-2nd
Paper-VI, unit-1st
Bilinear transformation. (1)

# Some elementary transformation

(I) Translation $\omega = z + \alpha$

A($\alpha$)

Let $z = x + iy$
$\alpha = a + ib$
and $\omega = u + iv$

Every point P(z) in the z-plane is translated in the direction of the vector $\alpha$ through a distance $|\alpha|$

O P(z) R

Then $\omega = z + \alpha$

$\Rightarrow u + iv = (x+iy) + (a+ib) = (x+a) + i(y+b)$

$\Rightarrow u = x + a, \; v = y + b$.

$\therefore$ The image of the point $P(x,y)$ in the z-plane under the given transformation is the point $R(x+a, y+b)$ in the $\omega$-plane which means that every point in the z-plane is translated in the direction of the vector $\alpha$ through a distance $|\alpha|$

By this transformation the figures in the z-plane have the same shape, size and orientation in $\omega$-plane.

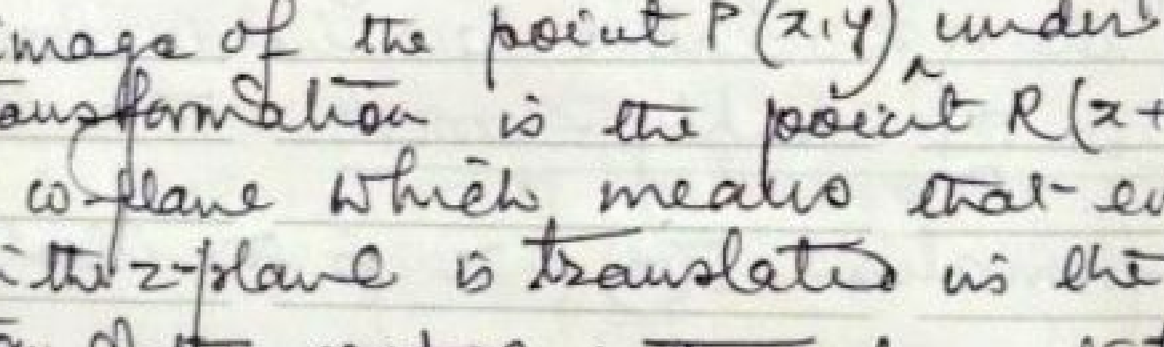
(II) Magnification $\omega = cz \; (c > 0)$

Stretching or Contracting

Let $\omega = u + iv$, $z = x + iy$

Then $\omega = cz \Rightarrow u + iv = c(x+iy)$

$\Rightarrow u = cx, \; v = cy$



To obtain the image of a point $P(x,y)$ in the z-plane ~~we take a point Q in OP~~ ~~$\exists$ $0<c<1$ or a point Q in OP produced~~ if $c>1$ such that $OQ = cOP$.



By this transformation the figures in the z-plane are stretched or contracted in the direction of z according $c>1$ or $0<c<1$.

III) Rotation and Magnification $\quad w = cz$ (c complex)

Let $w = \rho e^{i\phi}$, $c = ae^{i\alpha}$, $z = re^{i\theta}$.

Then $w = cz$

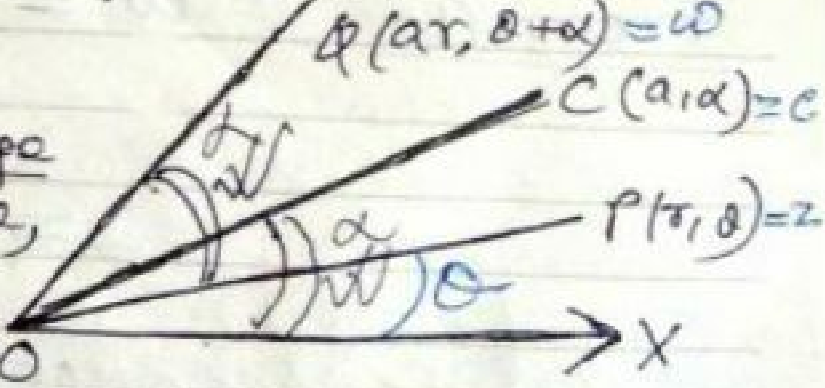
$\Rightarrow \rho e^{i\phi} = are^{i(\theta+\alpha)}$



$\Rightarrow \rho = ar, \ \phi = \theta+\alpha$

By this transformation the point $P(r,\theta)$ in the z-plane mapped into the point $Q(\rho,\phi)$ in the w-plane, where

$\rho = ar, \ \phi = \theta+\alpha$




Thus, in order to obtain the image of the point $P(r,\theta)$ in the z-plane, we rotate the radius vector OP through an angle $\alpha = \arg c$ and then magnify OP in the ratio $|c|:1$. Hence Q will represent w provided $\angle QOP = \angle COX = \arg c$ and $OQ = OC \cdot OP$